

SUBIECTUL I

1. $x^2 - 2x - 3 = 0$ (2pct); $\Delta = 16$ (1pct); $x_1 = 3$ (1pct); $x_2 = -2$ (1pct)
2. $(2 + \sqrt{5})^2 = 9 + 4\sqrt{5}$ (2pct); $E = 9 + 4\sqrt{5} - 4(2 + \sqrt{5}) - 2$ (1pct); $E = -1$ (2pct)
3. $\cos 120^\circ = -\frac{1}{2}$ (2pct); $\sin^2 60^\circ = \frac{3}{4}$ (2pct); finalizare $\frac{1}{4}$ (1pct).
4. $f(x) = 0$ (1pct); $x^2 - 2x - 3 = 0$ (1pct); $x_1 = 3, x_2 = -1$ (2pct); intersecțiile sunt $A(3, 0), B(-1, 0)$ (1pct).
5. Forma ecuației (2pct); înlocuiri, răspunsul $x - y - 3 = 0$ (3pct).
6. $\log_2 8 = 3$ (1pct); $C_7^2 = 21$ (1pct); $a_4 = 57$ (3pct).

SUBIECTUL II

1. a) $\det A = 1$ (5pct)

b) $B = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (2pct); $B^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (3pct)

c) $\det A \neq 0 \Rightarrow \exists A^{-1}$ (1pct); $A^t = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ (1pct);

$$A^* = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix} \text{ (2pct) } A^{-1} = A^* \text{ (1pct)}$$

2. a) verificarea: (5pct);

b) $(x \circ y) \circ z = (x \circ y - 2) \circ (z - 2) = (x - 2)(y - 2)(z - 2)$
 $x \circ (y \circ z) = (x - 2) \circ (y \circ z - 2) = (x - 2)(y - 2)(z - 2)$ (2pct+2pct)

finalizare (1pct).

c) $x \circ x = (x-2)^2 + 2$ (1pct); presupunem $\underbrace{x \circ x \circ \dots \circ x}_{k\text{ori}} = (x-2)^k + 2$;

$\underbrace{x \circ x \circ \dots \circ x}_{k+1\text{ori}} = (\underbrace{x \circ x \circ \dots \circ x}_{k\text{ori}}) \circ x = (x-2)^{k+1} + 2$, deci $\underbrace{x \circ x \circ \dots \circ x}_{n\text{ori}} = (x-2)^n + 2$ (2pct);

$\underbrace{(x \circ x \circ \dots \circ x) = (x-2)^{2013} + 2}_{2013\text{ori}}$ (1pct); $(x-2)^{2013} + 2 = 3$ deci $(x-2)^{2013} = 1$ deci

$x-2=1, x=3$ (1pct).

SUBIECTUL III

1. a) $f'(x) = 2x + 3 + \frac{1}{x}$ (2pct+2pct+1pct)

b) $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = f'(2)$ (2pct); $= 4 + 3 + \frac{1}{2}$ (2pct); $= \frac{15}{2}$ (1pct).

c) $f''(x) = 2 - \frac{1}{x^2} = \frac{2x^2 - 1}{x^2}$ (1pct); $f''(x) = 0 \Leftrightarrow 2x^2 - 1 = 0$ deci $x = \frac{\sqrt{2}}{2}$ (2pct); pe

intervalul $(0, \frac{\sqrt{2}}{2})$ este concavă, iar pe intervalul $(\frac{\sqrt{2}}{2}, \infty)$ este convexă (2pct).

2. a) $\int_0^1 f(x) dx = I_1 + I_2 + I_3$;

$I_1 = \int_0^1 x^2 e^x dx = \int_0^1 x^2 (e^x)' dx = x^2 e^x \Big|_0^1 - \int_0^1 (x^2)' e^x dx = e - 2 \int_0^1 x e^x dx$ (1,5pct);

$= e - 2 \int_0^1 x (e^x)' dx = e - 2[xe^x \Big|_0^1 - \int_0^1 e^x dx] = e - 2$ (1,5pct); $I_2 = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2}$ si

$I_3 = x \Big|_0^1 = 1$ (1pct); $\int_0^1 f(x) dx = e - 2 + \frac{1}{2} + 1 = \frac{2e-1}{2}$ (1pct).

b) $f(x) - x^2 e^x = x + 1$ (1pct); $g(x) = (x+1)^4$ (1pct); primitivele sunt

$G(x) = \int (x+1)^4 dx = \frac{(x+1)^5}{5} + C$ (3pct).

c) $\int f(x) dx = x^2 e^x - 2x e^x + 2e^x + \frac{x^2}{2} + x + C$ (2pct); $2x = t$ deci

$\int_0^1 f(2x) dx = \frac{1}{2} \int_0^2 f(t) dt = 2e^2 + 2$ (3pct)