Barem simulare bacalaureat matematica, filiera teoretica, matematica-informatica, 26.03.2013, GORJ.

SUBIECTUL I

1. $x^3 = y, y^2 + 7y - 8 = 0, y_1 = -8, y_2 = 1$ **(1pct);** $x^3 = -8 ... x \in \{-2, 1 \pm i\sqrt{3}\}$ **(2pct);**
   $x^3 = 1 ... x \in \{1, -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}\}$ **(2pct)**

2. $n^2 < n^2 + n < n^2 + 2n + 1$ **(2pct);** $n < \sqrt{n^2 + n} < n + 1$ **(2pct);** $\left[\sqrt{n^2 + n}\right] = n$ **(1pct)**

3. $1 + 3 + 5 + ... + (2n - 1) = n^2$ **;inducţie:verificare (1pct); demonstraţie (2pct);**

$\sqrt{1 + 3 + 5 + ... + (2n - 1)} = n \in N$ **(2pct).**

4. **O, centrul paralelogramului** $\Rightarrow \overrightarrow{MA} + \overrightarrow{MC} = 2\overrightarrow{MO}$ **(2pct);** $\overrightarrow{MB} + \overrightarrow{MD} = 2\overrightarrow{MO}$ **(2pct);**
   finalizare (1pct).
5. **Tabel de variaţie(1pct); scriere A (2pct); scriere B (2pct)**
6. **Condiţie** $n \geq 2$ **(0,5pct);** $C_n^2 = \frac{n(n-1)}{2}$ **(1pct);** $A_n^2 = n(n-1)$ **(1pct);** $n^2 - n - 20 = 0$ **şi rezolvarea ecuaţiei (2pct); soluţie n=5 (0,5pct).**

SUBIECTUL II

1. **a)** $B^2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ **(2pct);** $B^3 = I_3$ **(3pct)**

   b) SOLUŢIA I $B^3 = I_3 \Rightarrow BB^2 = B^2B = I_3 \Rightarrow B^{-1} = B^2$ **(5pct)**

   SOLUŢIA II $\det(B) = 1$ **(1pct);** B^t **(1pct);** B^* **(2pct);** B^{-1} **(1pct)**

   c) $A = \begin{pmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{pmatrix}$ **(1pct);** $\det(A) = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$ **(2pct);**

$(a+b+c)\det(A)=(a+b+c)^2\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]\geq 0$ **(egalitatea 1pct, inegalitatea 2pct).**

2. a) $\begin{matrix} x_1+x_2+x_3+x_4=6 \\ x_1x_2+x_1x_3+...x_3x_4=13 \end{matrix}$ **(2pct);**

$x_1^2+x_2^2+x_3^2+x_4^2=(x_1+x_2+x_3+x_4)^2-2(x_1x_2+x_1x_3+...x_3x_4)$ **(2pct); finalizare =** 10 **(1pct);**

b) SOLUŢIA I

$\begin{matrix} f(-1)=0 \\ f(2)=0 \end{matrix}$ **(1pct);** $-a+b=-20$**;** $2a+b=-20$ **(3pct);** $a=0,b=-20$ **(1pct)**

SOLUŢIA II

$(x+1)(x-2)=x^2-x-2$ **(1pct); efectuarea împărţirii (2pct); restul=polinom nul, scrierea sistemului (1pct); rezolvarea sistemului** $a=0,b=-20$ **(1pct).**

c) $x_1=x_2,x_3=x_4$ **deci primele două relatii Viete devin** $x_1+x_3=3,x_1^2+4x_1x_3+x_3^2=13$ **(1pct);**

$x_1=1,x_2=2$ **(2pct);** $f(1)=0,\ f(2)=0$ **(0,5pct); scrierea şi rezolvarea sistemului,** $a=-12,b=4$ **(1,5pct).**

SUBIECTUL III

1. a) $f^{'}(x)=ne^{nx}+3x^2-2x+1$ **(2pct);** $\lim\limits_{x\to 0}\frac{f(x)-1}{x}=f^{'}(0)=n+1$ **(3pct)**

b) $f^{'}(x)=ne^{nx}+3(x-\frac{1}{3})^2+\frac{2}{3}>0,\forall x\in R$**, f strict crescătoare deci injectivă (2pct);**

$\lim\limits_{x\to-\infty}f(x)=-\infty$ **(1pct);** $\lim\limits_{x\to+\infty}f(x)=+\infty$ **(1pct) , f continuă (are proprietatea lui Darboux), deci** $f(R)=R$ **deci este surjectivă (1pct).**

c) Derivata a doua este $f^{''}(x)=n^2e^{nx}+6x-2$ **(2pct); de exemplu, pt.** $6x-2>0\Rightarrow(\frac{1}{3},+\infty)$ **este un interval pe care funcţia este convexă (3pct).**

2. a) $I_1=\int_0^1\frac{x}{x^2+3x+2}dx=\frac{1}{2}\int_0^1\frac{2x+3-3}{x^2+3x+2}dx=\frac{1}{2}\int_0^1\frac{2x+3}{x^2+3x+2}dx-\frac{3}{2}\int_0^1\frac{1}{(x+1)(x+2)}dx=$ **(2pct)**

$$=\frac{1}{2}\ln(x^2+3x+2)\Big|_0^1-\frac{3}{2}\int_0^1\frac{1}{1+x}dx+\frac{3}{2}\int_0^1\frac{1}{2+x}dx$$ **(2pct)=** $\ln\frac{9}{8}$ **(1pct).**

b) $I_{n+2}+3I_{n+1}+2I_n=\int_0^1\frac{x^{n+2}}{x^2+3x+2}dx+\int_0^1\frac{3x^{n+1}}{x^2+3x+2}dx+\int_0^1\frac{2x^n}{x^2+3x+2}dx$ **(1pct)**

$=\int_0^1\frac{x^{n+2}+3x^{n+1}+2x^n}{x^2+3x+2}dx$ **(2pct)** $=\int_0^1\frac{x^n(x^2+3x+2)}{x^2+3x+2}dx$ **(1pct)** $=\int_0^1 x^n dx=\frac{1}{n+1}$ **(1pct)**

c) $nI_n=\int_0^1\frac{nx^n}{x^2+3x+2}dx=\int_0^1 nx^n(\frac{1}{x+1}-\frac{1}{x+2})dx=\int_0^1(x^n)'x(\frac{1}{x+1}-\frac{1}{x+2})dx$ **(1pct)**

$=x^{n+1}(\frac{1}{x+1}-\frac{1}{x+2})\Big|_0^1-\int_0^1 x^n[\frac{1}{(x+1)^2}-\frac{1}{(x+2)^2}]dx$

$=\frac{1}{2}-\frac{1}{3}-\int_0^1[\frac{x^n}{(x+1)^2}-\frac{x^n}{(x+2)^2}]dx$ **(1,5pct)**

$0\le x^n\le 1\Rightarrow 0\le\frac{x^n}{(x+1)^2}\le x^n\Rightarrow 0\le\int_0^1\frac{x^n}{(x+1)^2}dx\le\frac{1}{n+1}$ **cu teorema "cleşte"**

$\lim_{n\to\infty}\int_0^1\frac{x^n}{(x+1)^2}dx=0$ **(2pct) Analog** $\lim_{n\to\infty}\int_0^1\frac{x^n}{(x+2)^2}dx=0$ **deci** $\lim_{n\to\infty}nI_n=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$ **(0,5pct)**

ALTFEL

$x\in[0,1]\Rightarrow x^{n+1}\le x^n\Rightarrow I_{n+1}\le I_n$ **(1pct)**

$\frac{1}{n+1}=I_{n+2}+3I_{n+1}+2I_n\le I_n+3I_n+2I_n=6I_n$ **deci** $\Rightarrow I_n\ge\frac{1}{6(n+1)}$ **(2pct)**

$\frac{1}{n-1}=I_n+3I_{n-1}+2I_{n-2}>I_n+3I_n+2I_n=6I_n$ **deci** $\Rightarrow I_n\le\frac{1}{6(n-1)}$ **(1pct)**

Cu teorema "cleste" rezulta cerinta (1pct).

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